Requirements on orbitography for space clock missions

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Abstract— **In this paper, we study in detail the requirements on orbitography compatible with operation of next generation space clocks at the required uncertainty, and based on a completely relativistic model. We show that the required accuracy goal can be reached with relatively modest constraints on the orbitography of the space clock, much less stringent than expected from "naive" estimates using the example of the ACES mission. Our results are generic to all space clocks and represent a significant step towards the generalised use of next generation space clocks in fundamental physics, geodesy, and time/frequency metrology.**

I. Introduction

Over the last decade of the 20^{th} century and the first few years of the 21^{st}, the uncertainty of atomic clocks has decreased by over two orders of magnitude, passing from the low 10^{-14} to the below 10^{-16}, in relative frequency [2], [5], [7]. This rapid evolution is essentially due to recent technological breakthroughs (laser cooling and trapping of atoms and ions), which allow very effective control and reduction of the movement of the atoms and correspondingly long interrogation times. Atomic fountain microwave clocks use freely falling laser cooled atoms and were the first to reach uncertainties below 10^{-15} some of them being at present in the low 10^{-16} range. They are now closely followed, and even surpassed, by trapped ion and neutral atom optical clocks, the best of which show uncertainties below 10^{-16}.

Space applications in fundamental physics, geodesy, time/frequency metrology, navigation, etc... are among the most promising for this new generation of clocks. Onboard terrestrial or solar system satellites, their exceptional frequency stability and accuracy make them a prime tool to test the fundamental laws of nature, and to study the Earth's and solar system gravitational potential and its evolution. In the longer term, they are likely to provide the primary time reference for the Earth, as clocks on the ground will be subject to the less accurate knowledge of the geopotential on the surface [12].

For example, when comparing a clock on a low Earth orbiting satellite ($\approx$ 1000 km altitude) to one on the ground they display a difference of $\approx 10^{-10}$ in relative frequency due to the relativistic gravitational frequency shift. Measuring that difference with 10^{-17} uncertainty would allow a test of the gravitational frequency shift to a few parts in 10^{7} or equivalently, a determination of the potential difference between the clocks at the 10 cm level. The latter would contribute significantly to the knowledge of the geopotential and related applications in geophysics, representing the first realisation of relativistic geodesy [3], [9] where the fundamental observable is directly the gravitational potential via the relativistic redshift.

From the above it is obvious that next generation space clocks at the envisaged uncertainty level require a fully relativistic analysis and modelling, not only of the clocks (in space and on the ground) but also of the time/frequency transfer method used to compare them [1], [6], [8], [12], [4], [11]. Indeed, a highly accurate space clock is of limited use unless it can be compared to ground clocks using a method that does not degrade the overall uncertainty, and unless the behaviour of the clocks as a function of their positions and velocities can be modelled with sufficient accuracy. As an example, simple order of magnitude estimates of the relativistic gravitational frequency shift show that an 1 m error on the position of the clocks leads to an error of $\approx 10^{-16}$ in the determination of their frequency difference. Similarly when using an one-way system (GPS like) for the time transfer an 1 m position error leads to an error of $\approx 3 \times 10^{-9}$ s in the synchronisation ie. $\geq 10^{-14}$ in relative frequency over one day.

In this paper, we study in more detail the requirements on orbitography compatible with operation of next generation space clocks at the required uncertainty, and based on a completely relativistic model. We use the example of the ACES (Atomic Clock Ensemble in Space) mission, an ESA-CNES project to be installed onboard the ISS (International Space Station) in 2014. It consists of two atomic clocks and a two-way time transfer system (microwave link, MWL) with an overall uncertainty goal of 1 part in 10^{16} after ten day integration (see section III for more details). We show that the required accuracy goal can be reached with relatively modest constraints on the orbitography of the space clock of $\approx$ 10 m in position, which is about an order of magnitude less stringent than expected from "naïve" estimates ($\approx$ 1 m, see above). This is due to first order cancellation between the velocity and position part of the orbitography error in the determination of the relativistic frequency shift of the space clock, and to the use of a two-way time transfer system (MWL) which leads to first order cancellation of the position errors in

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the clock comparison (see section V). Our results are generic to all space clocks (not limited to the ACES mission) and represent a significant step towards the generalised use of next generation space clocks in fundamental physics, geodesy, and time/frequency metrology, as they show that the constraints on the orbitography of the space clock are much less stringent than previously thought. In sections II and III we briefly describe the ACES mission and the relativistic model used for the clocks and the time transfer, followed by a description of a model for the orbitography error to be expected onboard the ISS, based on measurements using an in situ GPS receiver (section IV). Our main results are the calculation of the effect of that error on the determination of the relativistic frequency shift of the clocks and on the time transfer (MWL) for the ACES mission (section V), where we show that the mission objectives can be achieved with the relatively modest orbitography of section IV and, more generally, calculate the overall requirements on orbitography for the ACES mission. We discuss those results and conclude in section VI.

II. The ACES mission

The ACES project led by the CNES and the ESA aims at setting up on the ISS several highly stable clocks around 2014. The ACES payload consists of two clocks : a cold atom clock PHARAO developed by CNES and a hydrogen maser (SHM developed by Neuchâtel observatory) together with a microwave communication link. The frequency stability of PHARAO onboard the ISS is expected to be better than 10^{-13} for one second, $3 \cdot 10^{-16}$ over one day and $1 \cdot 10^{-16}$ over ten days, with an accuracy goal of $1 \cdot 10^{-16}$ in relative frequency.

The ACES mission has as objectives :

- to operate a cold atom clock in microgravity with a 100 mHz bandwidth,
- to compare the high performances of the two atomic clocks in space (PHARAO and SHM) and to obtain a stability of $3 \cdot 10^{-16}$ over one day,
- to perform time comparisons between the two space clocks and ground clocks,
- to carry out tests of fundamental physics such as a gravitational redshift measurement and to search for a potential speed of light anisotropy and a possible drift of the fine structure constant α.
- to perform precise measurements of the Total Electron Content (TEC) in the ionosphere, the tropospheric delay and the Newtonian potential.

The time transfer is performed using a micro-wave two-way system, called Micro-Wave Link (MWL). An additional frequency is added in order to measure and correct the ionospheric delay at the required level. It uses carriers of frequency 13.5, 15.0 et 2.25 GHz modulated by pseudo random codes. Moreover it has four channels that allow four ground stations to be compared with the ISS clock at the same time.

The Micro-Wave Link (MWL) is an important part of the ACES mission. According to the mission specifications it has to synchronize two atomic clocks with a time stability of ≤ 0.3 ps over 300 s, ≤ 7 ps over one day, and ≤ 23 ps over 10 days. The performance of this link is a key issue since it will perform high precision time comparisons without damaging the high performances of the clocks.

For our purposes we express the above requirements for the MWL in a simplified form by the temporal Allan deviation $(\sigma_x(\tau))$:

$$\sigma_x(\tau) = 5.2 \cdot 10^{-12} \cdot \tau^{-\frac{1}{2}} \qquad (1)$$

for one single passage (for integration times τ lower than 300 s) and by

$$\sigma_x(\tau) = 2.4 \cdot 10^{-14} \cdot \tau^{\frac{1}{2}} \qquad (2)$$

for longer integration times (for integration times τ greater than 300 s).

The temporal Allan deviation can be related to frequency instability as expressed by the modified Allan deviation $Mod\sigma_y(\tau)$ by

$$Mod\sigma_y(\tau) = \frac{\sqrt{3}\ \sigma_x(\tau)}{\tau}. \qquad (3)$$

We take (1) and (2) as our upper limits for the calculation of all perturbing effects in the following sections.

III. Relativistic model for clocks and time transfer of ACES

In a general relativistic framework each clock produces its own local proper time, in our case τ^g and τ^s for the ground and space clocks respectively.

In order to model signal propagation between the ground and the space stations, we use a non-rotating geocentric space-time coordinate system. Thus $t = x_0/c$ is the geocentric coordinate time, $\overrightarrow{x} = (x_1, x_2, x_3)$ are the spatial coordinates, where c is the speed of light in vacuum (c = 299792458 $m.s^{-1}$). We denote $U(t, \overrightarrow{x})$ as the total Newtonian potential at the coordinate time t and the position $\overrightarrow{x}$ with the convention that $U \geq 0$ [10]. In these coordinates, the metric is given by an approximate solution of Einstein's equations valid for low velocity and potential ($\frac{U}{c^2} << 1$ and $\frac{v^2}{c^2} << 1$):

$$ds^2 = -(1 - \frac{2U(t, \overrightarrow{x})}{c^2})c^2 dt^2 + (1 + \frac{2U(t, \overrightarrow{x})}{c^2})d\overrightarrow{x}^2 \qquad (4)$$

where higher order terms can be neglected for our purposes [12].

In this system, each emission or reception event (at the antenna phase center) is identified by its own coordinate time t_i (figure (1)) and a coordinate time interval is defined by $T_{ij} = t_j - t_i$.

The f_1 frequency signal is emitted by the ground station at the coordinate time t_1 and received by the space station at t_2. The f_2 and f_3 frequency signals are emitted from the space station at t_3 and t_5, and received at the ground station at t_4 and t_6. The third frequency is added to measure the TEC in the ionosphere which allows the correction of the ionospheric delay.

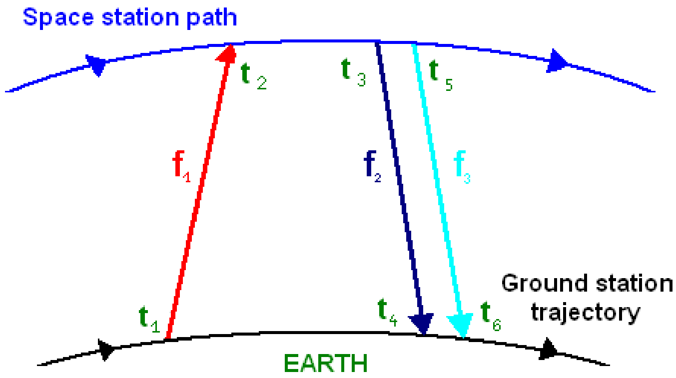

Fig. 1. MWL principle

The MWL is characterized by its continuous way of emission. It measures the time offsets between the locally generated signal and the received one. It provides three measurements (or observables) of the code (one on the space station, two on the ground) and three measurements of the phase of the carrier frequency at a sampling rate of one Hertz. Two identical sequences of code, referenced at the respective local time, are produced on ground and space stations respectively. An observable is a measurement of the local time interval between production τ_p of a segment of code and the arrival τ_a of the same segment, where τ_p and τ_a are the center time of segment produced and received at the station. The observable is labelled with the arrival proper time τ_a.

For instance, the upwards signal observable is:

$$\Delta\tau^s(\tau^s_a) = \tau^s_p - \tau^s_a = \tau^g_p - \tau^s_a$$

the last identity comes from the fact that two identical segments of code are produced at the same local time : $\tau^g_p = \tau^s_p$

Thus $\Delta\tau^s(\tau^s(t_2)) = \tau^g(t_1) - \tau^s(t_2)$

Considering the experimental uncertainties of the ACES mission (see figure 1), we will neglect any terms in the relativistic model that, when maximised, lead to corrections of less than $3 \cdot 10^{-13}$ s in time. Numerical applications are necessary to evaluate which terms can be neglected. For this purpose, we consider the International Space Station with a circular trajectory at an altitude of 400 km in a plane inclined by 51.6^o with respect to the equatorial plane. It has a velocity $v_s = 7.,7 \cdot 10^3\ m.s^{-1}$ in a gravitational potential of $U_s/c^2 = 6.,5 \cdot 10^{-10}$. The ground station has a velocity $v_g = 465\ m.s^{-1}$ at a gravitational potential of $U_g/c^2 = 6.9 \cdot 10^{-10}$.

The tropospheric delay Δ^{tropo} is considered as independent of the frequency of the signal, with a slow variation with time and an amplitude of less than 100 ns.

We assume the density of electrons in the ionosphere N_e is less than $2 \cdot 10^{12}$ electrons/m^3. The ionospheric delay Δ^{iono} is maximum for the f_3 frequency signal with an amplitude of about 2.3 ns.

The relation between the proper time τ and the coordinate time t is given to sufficient accuracy, e.g. by Blanchet and al. (2001) [4].

$$\frac{d\tau}{dt} = 1 - \left(\frac{U(t,\overrightarrow{x})}{c^2} + \frac{v^2(t)}{2c^2}\right) + O(c^{-4}) \tag{5}$$

Higher order terms of equation [4] have negligible effects at the projected uncertainty of $1 \cdot 10^{-16}$ in relative frequency of the ACES clocks [12]. Note however, that some care has to be taken when evaluating the Newtonian potential $U(t,\overrightarrow{x})$ in [4] for the ground or space clock [12].

The ACES mission aims at obtaining the variation of the desynchronisation between ground and space clocks with time, that is to say, the function $\tau^g(t) - \tau^s(t)$. It is evaluated by combining the measurements performed on the ground and onboard the space station and a precise calculation of the signal propagation times. In order to be able to control T_{23} (see below), we combine two measurements $\Delta\tau^s(\tau^s(t_2))$ and $\Delta\tau^g(\tau^g(t_4))$ but with $\tau^s(t_2) \neq \tau^g(t_4)$. Then the expression of desynchronisation reads

$$\begin{aligned}\tau^g(t_a) - \tau^s(t_a) = {} & \frac{1}{2}(\Delta\tau^s(\tau^s(t_2)) - \Delta\tau^g(\tau^g(t_4)) \\ & + T_{12} - T_{34} \\ & - \int_{t_1}^{t_2}(\frac{U(t,\overrightarrow{x_g})}{c^2} + \frac{v_g^2(t)}{2c^2})dt \\ & + \int_{t_3}^{t_4}(\frac{U(t,\overrightarrow{x_s})}{c^2} + \frac{v_s^2(t)}{2c^2})dt)\end{aligned} \tag{6}$$

where $t_a = \frac{t_2+t_4}{2}$, and where $\Delta\tau^s(\tau^s(t_2))$ and $\Delta\tau^g(\tau^g(t_4))$ are the observables respectively from the ground and onboard the satellite at the coordinate times t_2 and t_4, and where we have neglected non-linearities of $\tau^g(t)$ and $\tau^s(t)$ over the interval $t_4 - t_2$ (few milliseconds). The integral terms result from proper time to coordinate time transformations. They are small corrections of order 10^{-12} s to the desynchronisation $\tau^g(t_a) - \tau^s(t_a)$.

However it is the derivative with respect to the coordinate time t of the relation (6) which has to be studied for applications such as gravitational redshift or geodesy. Actually it has to be compared with the next relation obtained from equation (5).

$$\frac{d\tau^g}{dt} - \frac{d\tau^s}{dt} = \frac{1}{c^2} \cdot \left(U(t,\overrightarrow{x_s}) - U(t,\overrightarrow{x_g}) + \frac{v_s^2(t)}{2} - \frac{v_g^2(t)}{2}\right) \tag{7}$$

We note that any constant term appearing in the desynchronisation expression (6) will have no effect on the final result (7) because of the derivation.

In (6) the difference $T_{12} - T_{34}$ needs to be calculated from the knowledge of satellite and ground positions and velocities (orbitography).The difference $T_{12} - T_{34}$ of upward and downward signals at f_1 and f_2 allows to eliminate to first order delaying and restraining factors such as range (D/c), troposphere or Shapiro effects. Due to the asymmetry of the paths, that cancellation is not perfect, and there are some terms left which depend on orbitography as well as on the coordinate time interval T_{23} elapsed between reception and emission at the phase centre of the MWL antenna onboard the ISS. The aim of this work is to estimate with a simple orbital model, which levels of accuracy on orbitography and calibration of internal delays (knowledge of T_{23}) are required to reach the expected performances. For that purpose only the leading terms are required ie.

$$T_{12} - T_{34} = 2\frac{\overrightarrow{D}(t_4).\overrightarrow{v_g}(t_4)}{c^2} + \frac{\overrightarrow{D}(t_4).\overrightarrow{\Delta v}(t_4)}{c \cdot D(t_4)} T_{23} + O(\frac{1}{c^3}). \quad (8)$$

where $\overrightarrow{D}(t) = \overrightarrow{x}_s(t) - \overrightarrow{x}_g(t)$, $D(t) = ||\overrightarrow{D}(t)||$ and $\overrightarrow{\Delta v}(t) = \overrightarrow{v}_g(t) - \overrightarrow{v}_s(t)$.

In summary, a reliable orbitography is required for two main reasons. On one hand to calculate precisely the corrections in equations (8). On the other hand, to evaluate correctly the terms on the right hand side of equation (7) ie. the second order Doppler and gravitational redshifts.

In addition, we also need a precise knowledge of the time interval T_{23}, (ie. of the onboard internal delays) in order to be able to calculate the corresponding terms in (8) with sufficient accuracy.

Equation (8) together with equation (7) for the gravitational redshift, is sufficient to derive the maximum allowed uncertainties on orbitography and internal delays in order to stay below the limits given by (1) and (2).

IV. Orbit determination error model

Now we investigate the effects of trajectory knowledge on the accuracy and the stability of the time transfer (see equation (8)) and on the estimation of the relativistic correction of the clock (see equation (7)).

For the time transfer (8) we have to consider the position of the antenna phase center, but it is the clock reference point trajectory which is important for equation (7). The trajectories of the antenna phase center and of the clock reference point are defined by the trajectory of ISS center of mass (orbit determination error), and a geometrical offset (vector ISS center of mass - reference point) which depends on the attitude and on the geometry of the ISS (on position errors in the ISS frame).

The differences between true and computed (using orbit determination and attitude) trajectories of the ISS center of mass have very specific structures. For example an eccentricity error gives no long term effects, but periodic errors can be important and the radial, along track and velocity errors are correlated. This means that position and velocities are not independent, and if possible, this has to be taken into account for a performance evaluation.

For weak eccentricity orbits, errors of the ISS center of mass position are given by the Hill model (or the Clohessy-Wiltshire model) which is an expansion of uncertainties with respect to a reference circular orbit. This error model takes into account the correlation between all orbitographic parameters. It gives their expressions in the local orbital frame ($\overrightarrow{R}$, $\overrightarrow{T}$ et $\overrightarrow{N}$) defined with $\overrightarrow{R}$ the unit vector between the Earth's center and the space station, $\overrightarrow{N}$ orthogonal to $\overrightarrow{R}$ and the inertial velocity and where $\overrightarrow{T}$ is orthogonal to $\overrightarrow{R}$ and $\overrightarrow{N}$. Then the position uncertainties along radial, tangential and normal axis are given as follows :

$$\begin{aligned}
\text{radial axis : } \delta R(t) &= \frac{1}{2}X \cdot \cos(\omega t + \varphi_R) + c_R \\
\text{tangential axis : } \delta T(t) &= -X \cdot \sin(\omega t + \varphi_R) - \frac{3}{2}\omega \cdot c_R \cdot t + d_R \\
\text{normal axis : } \delta N(t) &= Y \cdot \cos(\omega t + \varphi_N)
\end{aligned} \quad (9)$$

where X, Y, c_R and d_R are amplitude coefficients, and where ω is the orbital pulsation. Surface accelerations errors are not taken into account, because this model corresponds to a local error of the adjusted trajectory and adapted for short arc length.

An ISS orbit determination using an onboard GPS receiver gives the orders of magnitude of these coefficients. It typically leads to X and Y lower than ten meters, that is to say, a ten meter bound on the tangential and normal axes and a five meter bound on the radial axis. For our purpose bias (d_R) plays no role and the linear term (c_R) depends on arc length. Basically the longer the observation duration is, the smaller this coefficient becomes.

The major feature of the error model is to take into account the error correlations in the orbital plane. For instance, a positive radial bias leads to a negative error on the tangential velocity: the satellite is delayed with respect to the reference orbit.

We note ($\overrightarrow{X}_a(t)$, $\overrightarrow{V}_a(t)$) and ($\overrightarrow{X'}_a(t)$, $\overrightarrow{V'}_a(t)$) respectively the true and computed trajectories of the antenna phase center, and ($\overrightarrow{X}_c(t)$, $\overrightarrow{V}_c(t)$) and ($\overrightarrow{X'}_c(t)$, $\overrightarrow{V'}_c(t)$) respectively the true and computed trajectories of the clock reference point. We also define ($\overrightarrow{X}_o(t)$, $\overrightarrow{V}_o(t)$) the true trajectory and the true velocity of the center of mass of the station. These five trajectories are expressed in non-rotating geocentrical frame (GCRS, Geocentric Celestial Reference System)[10].

Now we have to express the effects of station trajectory and time calibration uncertainties on time transfer and on gravitational redshift.

On one hand, according to the equation (6), the error in the time transfer is related with the uncertainties of $T_{12} - T_{34}$ entering in the desynchronisation . It can be obtained from the simplified equation (8) and is then dependent on the ground and space station trajectory knowledge, of the value of T_{23} and of the uncertainty on this parameter. As said before, a precise knowledge of the time interval T_{23} is related to the internal delay calibrations. The error on $T_{12} - T_{34}$ is

$$
\begin{aligned}
\delta\left(T_{12}-T_{34}\right) = {} & 2\frac{\delta\overrightarrow{D}.\overrightarrow{v_g}+\overrightarrow{D}.\delta\overrightarrow{v_g}}{c^2}+\frac{\overrightarrow{D}.\Delta\overrightarrow{v}}{c\cdot D}\delta T_{23} \\
& +\left(\frac{\delta\overrightarrow{D}.\Delta\overrightarrow{v}}{c\cdot D}+\frac{\overrightarrow{D}.\delta\Delta\overrightarrow{v}}{c\cdot D}-\frac{\overrightarrow{D}.\Delta\overrightarrow{v}}{c\cdot D}\frac{\delta D}{D}\right)T_{23}
\end{aligned}
\tag{10}
$$

If we suppose the uncertainty on ground station position is negligible with respect to the ISS position errors, the knowledge of the vector $\overrightarrow{D}$ is related to the uncertainty on the precision on the space station reference point which is the antenna phase center. Then we have $\delta\overrightarrow{D}=\overrightarrow{X_aX'_a}$.

The previous equation can be written as :

$$
\begin{aligned}
\delta\left(T_{12}-T_{34}\right) = {} & 2\frac{\overrightarrow{X_aX'_a}.\overrightarrow{v_g}}{c^2}+\frac{\overrightarrow{D}.\overrightarrow{\Delta v}}{c\cdot D}\delta T_{23} \\
& +[\frac{\overrightarrow{X_aX'_a}.\overrightarrow{\Delta v}}{c\cdot D}-\frac{\overrightarrow{D}}{c\cdot D}.\frac{d\overrightarrow{X_aX'_a}}{dt} \\
& -\frac{\overrightarrow{D}.\overrightarrow{\Delta v}}{c\cdot D}\frac{\|\overrightarrow{X_aX'_a}\|}{D}]T_{23}
\end{aligned}
\tag{11}
$$

On the other hand, the computation of the clock relativistic correction along a trajectory is defined by equation (7). It depends on the position and the velocity of the reference point, in this case the clock. We need to express the error on the reference point frequency - that is to say the frequency difference between the true clock position and the computed clock position - in order to compare its Modified Allan stability with the specifications.

The gravitational potential can be evaluated on a given trajectory with a sufficient precision using gravity models (eg. GRIM5 or EGM96). The error on the frequency shift at the clock position is given by:

$$
\begin{aligned}
\delta(\frac{d\tau}{dt})_{\overrightarrow{X_c}} &= (\frac{d\tau}{dt})_{\overrightarrow{X_c}}-(\frac{d\tau}{dt})_{\overrightarrow{X'_c}} \\
&= -\frac{1}{c^2}\left(U(t,\overrightarrow{X_c})-U(t,\overrightarrow{X'_c})+\frac{V_c^2-V_c'^2}{2}\right)
\end{aligned}
\tag{12}
$$

Using the fact that $\overrightarrow{X}_o$ is the solution of the differential equation

$$
\frac{d^2\overrightarrow{X}_o}{dt^2}=\overrightarrow{Grad}(U)+\overrightarrow{\Gamma}_S \tag{13}
$$

where $\overrightarrow{\Gamma}_S$ is the non gravitational acceleration, we obtain :

$$
\begin{aligned}
\delta(\frac{d\tau}{dt})_{\overrightarrow{X_c}} = {} & \frac{1}{c^2}[\frac{d}{dt}\left(\overrightarrow{V_o}.\overrightarrow{X_cX'_c}\right)+\frac{1}{2}(\frac{d\overrightarrow{X_oX_c}}{dt})^2 \\
& -\frac{1}{2}(\frac{d\overrightarrow{X_oX'_c}}{dt})^2-\overrightarrow{\Gamma}_S.\overrightarrow{X_cX'_c}]
\end{aligned}
\tag{14}
$$

In order to simplify the equation (14), we evaluate the order of magnitude of the different contributors appearing in this equation.

To investigate the importance of the term $\frac{\overrightarrow{\Gamma}_S.\overrightarrow{X_cX'_c}}{c^2}$, the drag has been modelled along a reference orbit of the ISS, for various altitudes. A period with important solar activity has been chosen in order to evaluate the worst case (see figure 2).

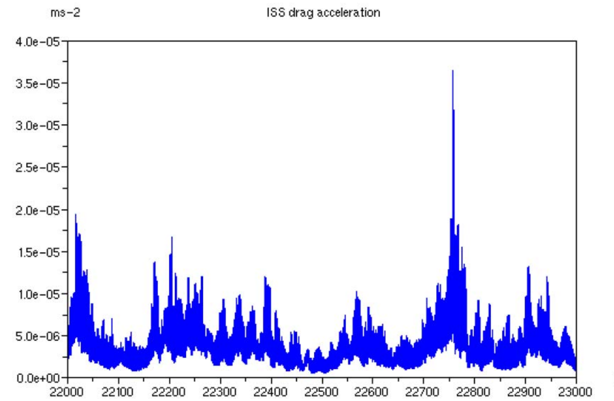

Fig. 2. Simulation of the non gravitational acceleration with time

To estimate its effect on formula (14), the acceleration has been multiplied by a 10 meter bias, or a 10 meter sinusoidal function at orbital period, corresponding to possible attitude and orbit errors effects of the ISS. The Allan variance stays below 10^{-21}, which is totally negligible here. This term has also no effect on the frequency accuracy. The effect of other surface accelerations like solar radiation pressure is also negligible.

The residual term of the second order Doppler shift $\frac{1}{2c^2}\left[(\frac{d\overrightarrow{X_oX_c}}{dt})^2-(\frac{d\overrightarrow{X_oX'_c}}{dt})^2\right]$ must be computed with the GCRS trajectories for attitude or orbital errors. The order of magnitude of these terms can be evaluated as $\frac{5(a\omega\cos(\omega t))^2}{2c^2}$ with ω the orbital pulsation and a a typical offset corresponding to the position in local orbital frame or to orbit errors. The corresponding Allan variance is bounded by $10^{-16}\cdot\tau^{-1}$ for a=10 meters. This effect is also negligible. However because of the power of two, this term does not have a zero mean. The magnitude of this frequency bias can be evaluated as equal to $1.7\cdot10^{-21}$. As far as the ACES mission is concerned, this effect can also be neglected.

The only important term for the performance evaluation is thus the along track term $\frac{1}{c^2}\frac{d}{dT}\left(\overrightarrow{V_o}.\overrightarrow{X_cX'_c}\right)$

So only the component of the clock error parallel to the velocity of the ISS plays a role. This can be understood considering for example a purely positif radial component. In this case we underestimate the gravitational potential but overestimatethe velocity (at constant ω), so the two cancel.

The scalar products of vectors can be evaluated in a local frame : for example it may be useful to study them in the local orbital frame ($\overrightarrow{R}$,$\overrightarrow{T}$,$\overrightarrow{N}$).

In this section, the errors on the time transfer and on the relativistic correction have been expressed in function of the trajectory knowledge through the equations (11) and (14). Moreover we have estimated that only the knowledge of the ISS center of mass position has an importance in the relativistic correction.

V. NUMERICAL RESULTS ON CLOCK COMPARISON

In this section we use the previously described error model to calculate realistic requirements for time transfer and gravitational frequency shift. For this purpose, we consider an ephemeris of ISS corresponding to the 20^{th} of May, 2005. It corresponds to a circular trajectory in a plane inclined by 51.6^{o} with respect to the equatorial plane. The orbitography of the ISS mass center is given in the rotating geocentrical frame and tagged in UTC. It has to be transformed to GCRS coordinates.

First we study the time transfer between the International Space Station and a ground station based in Toulouse, France $(43^{o}36'N, 1^{o}26'E)$. Actually, this station has been chosen as the master ground station of the ACES mission. Then we choose an ISS trajectory corresponding to one pass over Toulouse's ground station. All station parameters and their uncertainties have to be expressed in the same frame (ie. GCRS)

Errors are given in the local orbital frame, so we need to transform them to GCRS and determine the uncertainties (given by (9)) in this frame on position and on velocity parameters. These values will be used afterwards to calculate the maximum allowed uncertainties coming from equations (11) and (14).

For this purpose, we first consider the error equation (11) on the time transfer. We choose the signs of the independent parameters ($\overrightarrow{X_a X_a'}$, T_{23} and δT_{23}) so as to maximize the resulting temporal Allan deviation. The calculated deviations has to be compared with the MWL's specifications (eq. 1).

Assuming we have no error on T_{23} (ie. $\delta T_{23} = 0$ s), for all values of factor X (or Y), it is possible to determine the maximum value of the time interval T_{23} for which the temporal Allan deviation remains under the specifications. With numerous values of X, we calculate a bound which marks out two different areas. : the allowed uncertainties area in which each couple (X, T_{23}) gives a deviation staying under the specifications, and the prohibited area.

Figure (3) shows that, the smaller the time interval is, the less precise the space station position knowledge is required. This result provides a way to combine upwards and downwards signals in order to allow the maximum uncertainty on space station position to comply with the specifications. The most favorable situation to combine upwards and downwards signals is when the reception at the antenna phase center of the space station corresponds to the emission at the same place ie. $t_2 = t_3$. This way of combining signals is named the "Λ configuration". To work with parameter in the asymptotic area requires T_{23} to be under 10^{-6} s.

Then if we plot the maximum value of δT_{23} for all values of the factor X, there will appear two asymptotic values we cannot cross if we want to stay under the specifications. Basically a compromise between the knowledge of the space station trajectory and the precision of the internal delays

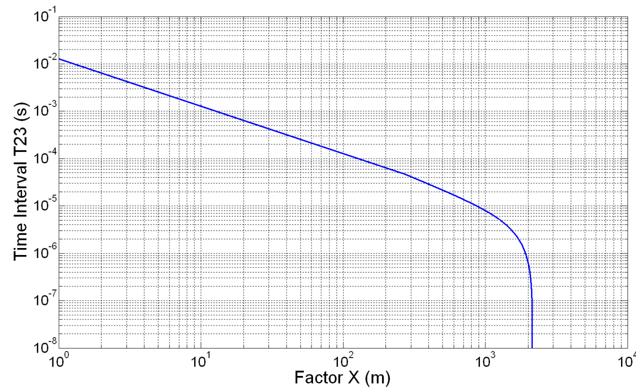

Fig. 3. Maximum allowed value of T_{23} as function of the scale factor X to comply with the specifications and with $\delta T_{23} = 0$

calibration must be achieved owing to the maximization of the Allan deviation. We will evaluate the maximum allowed errors on these two parameters if no other errors are present.

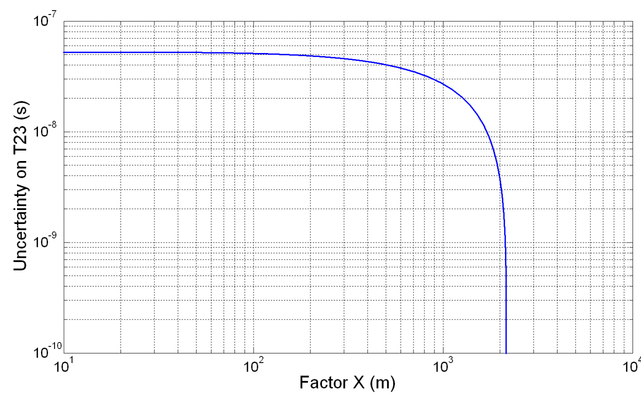

Fig. 4. Maximum allowed value of δT_{23} as function of the scale factor X to comply with the specifications and with $T_{23} = 0$

First we search for the asymptotic value of factor X which complies with the specifications for all phases (φ_R, φ_N) when we have no error on T_{23}. The asymptotic value for orbitography is obtained for X = 2150 m which corresponds to a 2.1 km error on the normal and tangential axes, and to an 1 km error on the radial axis.

The asymptotic value of the time calibration does not depend on orbitographic uncertainties. So it is independent of the phases φ_R and φ_N. We find that δT_{23} must stay under $5.2 \cdot 10^{-8}$ s.

Moreover the accuracy on the time transfer is not a problem because all the terms of the equation (11) have zero mean for one passage.

The requirements for several passes has also been investigated. In this case, the calculated deviations have to be compared to the specifications given by (2). The results of this work showed that the requirements on orbitography and time calibration are less stringent for several passes than for a single pass. Therefore if specifications are respected for a single pass, specifications for longer integration times are most likely also respected as the requirements on the uncertainty on T_{23} are less stringent in that case.

Now we evaluate requirements on orbitography considering the gravitational frequency shift. We search for the maximum value of X to comply with the gravitational frequency shift specifications (2). First term of (14) is evaluated with the error model (9), and its Allan deviation is calculated for different values of X. For integration time greater than one thousand seconds, these Allan deviations are independent of the phases φ_R and φ_N.

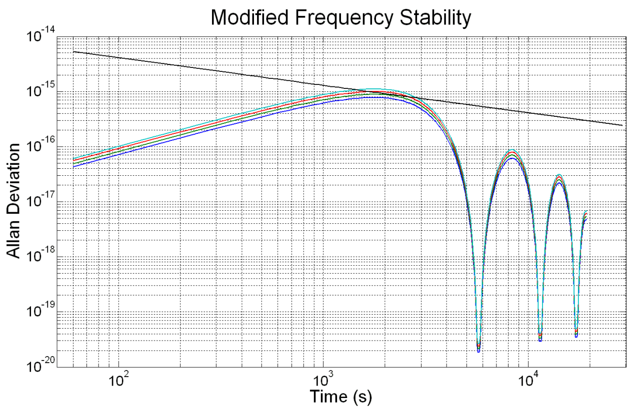

Fig. 5. Modified Allan deviations of the redshift error for X=14,16,18,20 m

Figure (5) shows that, if the factor X is equal to 16 m ie. if we have an eight meter error on the radial axis and sixteen meter error on the tangential axis, then we comply with the specifications.

The requirement on the factor Y is two orders of magnitude less stringent than on the factor X. This is mainly due to the projection of the position error along the ISS center of mass velocity (see equation (14)).

The accuracy requirement of ACES is 10^{-16} in relative frequency over ten days. The linear term on the tangential axis appearing in equations (9) when substituting into (14) could lead to an offset in frequency. This leads to a bound on this linear term:

$$\frac{3}{2}\omega c_R \Delta t < 1 \text{ km} \tag{15}$$

with $\Delta t = 10$ days.

This term will have no consequence on the accuracy.

VI. Conclusions

The formulations of time transfer and clock relativistic effects errors were described and applied on standard errors corresponding to orbit determination and geometry. We also evaluated the order of magnitude of the main effects.

Investigating the requirements for the ACES mission provides a way to combine upwards and downwards signals (the Λ configuration). Thus the requirements on orbitography and time calibration have been identified to reach stability specifications. They are summarized in table (I). The periodic error on the radial axis and on the tangential axis must stay respectively below eight and sixteen meters. The uncertainty on the normal axis is two orders of magnitude less stringent. Moreover the error on internal delays (δT_{23}) must not exceed fifty two nanoseconds. At last the requirement on the tangential drift (below one kilometer in ten days) is easily reached in order to comply with the accuracy specification (10^{-16} in relative frequency at ten days).

TABLE I
Requirements on parameters

X (m)	Y (m)	δT_{23} (ns)
16	2150	52

In conclusion the requirement on orbitography are significantly less stringent than the initial 'naive' estimate (one meter error for 10^{-16} in relative frequency) which is mainly due to partial cancellation between the gravitational redshift and the second order Doppler effect.

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